Ratio, Proportion, and Percentage

UNIT 18

Ratio and Proportion

Objectives *After studying this unit you should be able to*

- Write comparisons as ratios.
- Express ratios in lowest terms.
- Solve for the unknown term of a proportion.
- Substitute given numerical values for symbols in a proportion and solve for the unknown term.

The ability to solve practical machine shop problems using ratio and proportion is a requirement for the skilled machinist. Ratio and proportion are used for calculating gear and pulley speeds and sizes, for computing thread cutting values on a lathe, for computing taper dimensions, and for determining machine cutting times.

Description of Ratio

Ratio is the comparison of two like quantities.

Examples

1. Two pulleys are shown in Figure 18-1. What is the ratio of the diameter of the small pulley to the diameter of the larger pulley? All dimensions are in inches.

   The ratio is 3 to 5. Ans

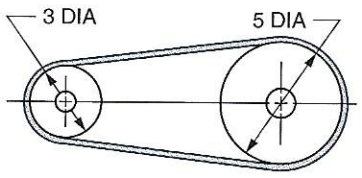


Figure 18-1

2. A triangle with given lengths of 3 meters, 4 meters, and 5 meters for sides a, b, and c is shown in Figure 18-2.

   a. What is the ratio of side a to side b?

      The ratio is 3 to 4. Ans

   b. What is the ratio of side a to side c?

      The ratio is 3 to 5. Ans

   c. What is the ratio of side b to side c?

      The ratio is 4 to 5. Ans

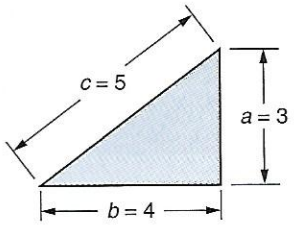


Figure 18-2

The *terms* of a ratio are the two numbers that are compared. **Both terms of a ratio must be expressed in the same units.**

Example Two pieces of bar stock are shown in Figure 18-3. What is the ratio of the short piece to the long piece?

The terms cannot be compared to a ratio until the 2-foot length is expressed as 24 inches.

The ratio is 11 to 24. Ans

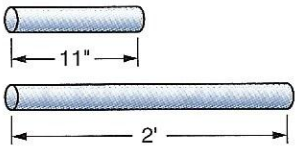


Figure 18-3

It is impossible to express two quantities as ratios if the terms have unlike units that cannot be expressed as like units. Inches and pounds as shown in Figure 18-4 cannot be compared as ratios.

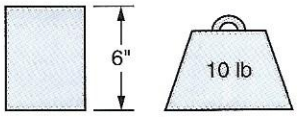


Figure 18-4

Expressing Ratios. Ratios are expressed in the following ways.

- With a colon between the two terms, such as 4:7. The ratio 4:7 is read as 4 to 7.
- With a division sign separating the two numbers, such as $4 \div 7$ or as a fraction, $\frac{4}{7}$.

Order of Terms

The terms of a ratio must be compared in the order in which they are given. The first term is the numerator of a fraction and the second is the denominator.

Examples

1. The ratio 1 to $3 = 1 \div 3 = \frac{1}{3}$ Ans

2. The ratio 3 to $1 = 3 \div 1 = \frac{3}{1}$ Ans

3. The ratio $x{:}y = x \div y = \frac{x}{y}$ Ans

4. The ratio $y{:}x = y \div x = \frac{y}{x}$ Ans

Expressing Ratios in Lowest Terms

Generally, a ratio should be expressed in lowest fractional terms.

Examples Each ratio is expressed in lowest terms.

1. $3{:}9 = \frac{3}{9} = \frac{1}{3}$ Ans
2. $40{:}15 = \frac{40}{15} = \frac{8}{3}$ Ans
3. $\frac{3}{8}{:}\frac{9}{16} = \frac{3}{8} \div \frac{9}{16} = \frac{3}{8} \times \frac{16}{9} = \frac{2}{3}$ Ans
4. $10{:}\frac{5}{6} = 10 \div \frac{5}{6} = \frac{10}{1} \times \frac{6}{5} = \frac{12}{1}$ Ans

Description of Proportions

A *proportion* is an expression that states the equality of two ratios.

Expressing Proportions. Proportions are expressed in the following ways.

- 3:4::6:8, which is read as "3 is to 4 as 6 is to 8."
- 3:4 = 6:8, which is read "the ratio of 3 to 4 equals the ratio of 6 to 8" or "3 is to 4 as 6 is to 8."
- $\frac{3}{4} = \frac{6}{8}$. This equation form is generally the way that proportions are written in practical applications.

A proportion consists of four terms. The first and the fourth term are called *extremes* and the second and third terms are called *means*.

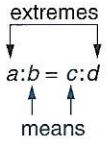
Examples

1. In the proportion 2:3::4:6

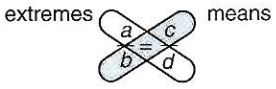
   2 and 6 are the extremes; 3 and 4 are the means. Ans

2. In the proportion $\frac{5}{6} = \frac{10}{12}$

   5 and 12 are the extremes; 6 and 10 are the means. Ans



or



In a proportion the product of the means equals the product of the extremes. If the terms are cross multiplied, their products are equal.

Examples

1. $\frac{3}{4} = \frac{6}{8}$

   Cross multiply, $\frac{3}{4} \times \frac{6}{8}$

   $3 \times 8 = 4 \times 6$

   $24 = 24$

2. $\frac{a}{b} = \frac{c}{d}$

   Cross multiply, $\frac{a}{b} \times \frac{c}{d}$

   $a \times d = b \times c$

   $ad = bc$

The method of cross multiplying is used in solving proportions that have an unknown term. You can check your answer by inserting it back in the original proportion.

Examples Solve for the value of x.

1. $\frac{3}{4} = \frac{x}{16}$

Cross multiply.

Divide both sides of the equation by 4.

Check.

$$\frac{3}{4} = \frac{x}{16}$$
$$4x = 3(16)$$
$$4x = 48$$
$$\frac{4x}{4} = \frac{48}{4}$$
$$x = 12 \quad \text{Ans}$$
$$\frac{3}{4} = \frac{x}{16}$$
$$\frac{3}{4} = \frac{12}{16}$$
$$\frac{3}{4} = \frac{3}{4} \quad \text{Ck}$$

2. $\frac{7}{x} = \frac{8}{15}$

$$8x = 7(15)$$
$$8x = 7(15)$$
$$\frac{8x}{8} = \frac{105}{8}$$
$$x = 13\frac{1}{8} \quad \text{Ans}$$

Check. $\frac{7}{x} = \frac{8}{15}$

$$\frac{7}{13\frac{1}{8}} = \frac{8}{15}$$
$$\frac{8}{15} = \frac{8}{15} \quad \text{Ck}$$

3. $\frac{x}{7.5} = \frac{23.4}{20}$

$$20x = 7.5(23.4)$$
$$20x = 175.5$$
$$\frac{20x}{20} = \frac{175.5}{20}$$
$$x = 8.775 \quad \text{Ans}$$

Solving by calculator: 7.5 ⊠ 23.4 ÷ 20 = 8.775 Ans

Check. $\frac{x}{7.5} = \frac{23.4}{20}$

$$\frac{8.775}{7.5} = \frac{23.4}{20} \quad \text{Ck}$$
$$1.17 = 1.17 \quad \text{Ck}$$

Application

Ratios

Express the following ratios in lowest fractional form.

1. 6:21 _____
2. 21:6 _____
3. 2:11 _____
4. 7:21 _____
5. 12":46" _____
6. 3 lb:21 lb _____
7. 13 mi:9 mi _____
8. 156 mm:200 mm _____
9. $\frac{2}{3}:\frac{1}{2}$ _____
10. $\frac{1}{2}:\frac{2}{3}$ _____

Related Ratio Problems

11. Length A in Figure 18-5 is 3 inches and length B is 2.5 feet. Determine the ratio of length A to length B in lowest fractional form. _____

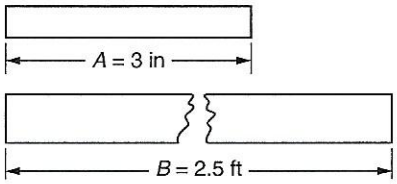


Figure 18-5

12. The diameters of pulleys E, F, G, and H, shown in Figure 18-6, are given in the table. Determine the ratios in lowest fractional form.

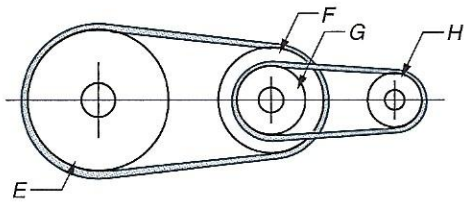


Figure 18-6

	DIAMETERS (inches)				RATIOS							
	E	F	G	H	$\frac{E}{F}$	$\frac{E}{G}$	$\frac{E}{H}$	$\frac{F}{G}$	$\frac{F}{H}$	$\frac{G}{H}$	$\frac{G}{E}$	$\frac{H}{F}$
a.	8	6	4	3								
b.	10	8	5	4								
c.	12	9	6	3								
d.	15	12	10	6								

13. Refer to the hole locations given for the plate in Figure 18-7. Determine the ratios in lowest fractional form. All dimensions are in millimeters.

a. Dimension A to dimension B. _____
b. Dimension A to dimension C. _____
c. Dimension C to dimension D. _____
d. Dimension C to dimension E. _____
e. Dimension D to dimension F. _____
f. Dimension F to dimension B. _____
g. Dimension F to dimension C. _____
h. Dimension E to dimension A. _____
i. Dimension D to dimension B. _____
j. Dimension C to dimension F. _____

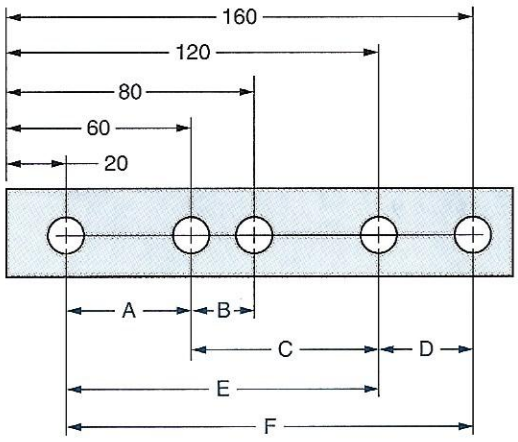


Figure 18-7

14. In Figure 18-8, gear A is turning at 120 revolutions per minute and gear B is turning at 3.6 revolutions per second. Determine the ratio of the speed of gear A to the speed of gear B. _____

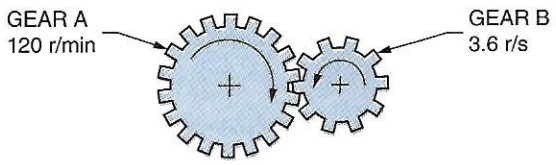


Figure 18-8

Proportions

Solve for the unknown value in each of the following proportions. Check each answer. Round the answers to 3 decimal places where necessary.

15. $\frac{x}{2} = \frac{6}{24}$ _____

16. $\frac{3}{A} = \frac{15}{30}$ _____

17. $\frac{7}{9} = \frac{E}{45}$ _____

18. $\frac{3}{13} = \frac{24}{y}$ _____

19. $\frac{15}{c} = \frac{5}{4}$ _____

20. $\frac{P}{27} = \frac{1}{3}$ _____

21. $\frac{6}{7} = \frac{15}{F}$ _____

22. $\frac{12}{H} = \frac{4}{25}$ _____

23. $\frac{T}{6.6} = \frac{7.5}{22.0}$ _____

24. $\frac{2.4}{3} = \frac{M}{0.8}$ _____

25. $\frac{4}{4.1} = \frac{8}{L}$ _____

26. $\frac{3.4}{y} = \frac{1}{9}$ _____

27. $\frac{24}{5} = \frac{3.2}{A}$ _____

28. $\frac{\frac{3}{8}}{N} = \frac{\frac{1}{2}}{4}$ _____

29. $\frac{\frac{3}{1}}{\frac{1}{4}} = \frac{5}{F}$ _____

30. $\frac{G}{\frac{1}{4}} = \frac{\frac{7}{8}}{\frac{3}{8}}$ _____

31. $\frac{\frac{7}{1}}{\frac{1}{8}} = \frac{x}{\frac{9}{16}}$ _____

32. $\frac{4}{R} = \frac{2.5}{12.5}$ _____

33. $\frac{11}{8} = \frac{E}{12}$ _____

34. $\frac{M}{12} = \frac{15}{9}$ _____

35. $\frac{6.08}{H} = \frac{5.87}{12.53}$ _____

36. $\frac{E}{7.53} = \frac{0.36}{1.86}$ _____

Related Proportion Problems

37. The proportion $\frac{A}{B} = \frac{C}{D}$ compares the sides of the two illustrated similar triangles like those in Figure 18-9. Determine the missing values in the table.

	A	B	C	D
a.	18"	4.5"		3"
b.	$6\frac{1}{2}$"	$1\frac{5}{8}$"	$4\frac{1}{2}$"	
c.	87.5 mm		75.0 mm	62.5 mm
d.		25.8 mm	20.6 mm	16.4 mm

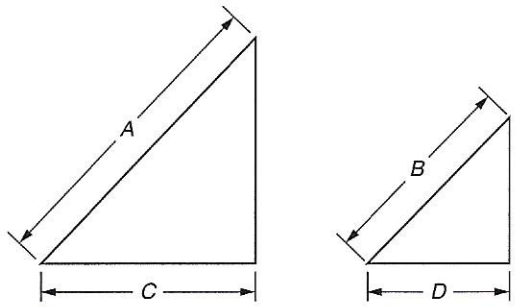


Figure 18-9

38. Where machine parts are doweled in position, it is good practice to extend the pin 1 to $1\frac{1}{2}$ times its diameter into the mating part as shown in Figure 18-10. Use the following proportion to determine the value of each unknown in the table. Round the answers to 3 decimal places where necessary.

$$\frac{N}{1} = \frac{L}{D}$$

where N = the number of times the pin extension is greater than the pin diameter

L = the length of the pin extension

D = the pin diameter

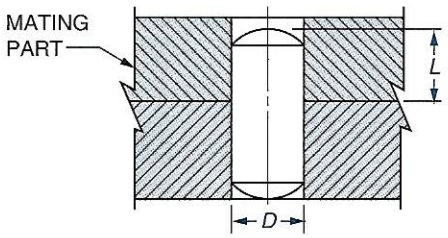


Figure 18-10

	N	D	L
a.	1.250	7.940 mm	
b.	$1\frac{1}{4}$	$\frac{1}{2}$"	
c.	$1\frac{1}{4}$		$\frac{3}{4}$"
d.	1.375		8.730 mm
e.	1.250	16.120 mm	

	N	D	L
f.	1.375		1.032"
g.	1.250	0.875"	
h.	1.500	3.680 mm	
i.	1.125		0.281"
j.	1.000	7.500 mm	

39. It is sometimes impractical to make engineering drawings full size. If the part to be drawn is very large or small, a scale drawing is generally made. The scale, which is shown on the drawing, compares the lengths of the lines on the drawing to the dimensions on the part. A scale on a drawing that states $\frac{1}{2}$" = 1" means" the drawing is one-quarter the size of the part. It is expressed as a ratio of 1:4 or $\frac{1}{4}$. A scale drawing that states 2" = 1"

means that the drawing is double the size of the part. It is expressed as a ratio of 2:1 or $\frac{2}{1}$.

The actual dimensions of a steel support are given in Figure 18-11. All dimensions are in inches. Using the proportion given, compute the lengths for each unknown in the table. Round the answers to 3 decimal places where necessary.

$$\frac{\text{numerator of scale ratio}}{\text{denominator of scale ratio}} = \frac{\text{drawing length}}{\text{part dimension}}$$

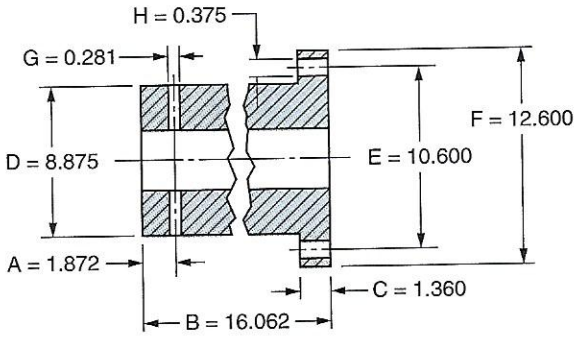


Figure 18-11

	Scale	Drawing Length
a.	$\frac{1}{2}" = 1"$	$B =$
b.	$4" = 1"$	$G =$
c.	$\frac{1}{4}" = 1"$	$B =$
d.	$2" = 1"$	$C =$
e.	$1\frac{1}{2}" = 1"$	$A =$
f.	$\frac{3}{4}" = 1"$	$E =$
g.	$3" = 1"$	$H =$
h.	$\frac{1}{8}" = 1"$	$F =$

	Scale	Drawing Length
i.	$\frac{1}{2}" = 1"$	$E =$
j.	$6" = 1"$	$G =$
k.	$\frac{3}{4}" = 1"$	$F =$
l.	$1\frac{1}{2}" = 1"$	$C =$
m.	$\frac{1}{2}" = 1"$	$F =$
n.	$3" = 1"$	$G =$
o.	$\frac{1}{4}" = 1"$	$B =$
p.	$2" = 1"$	$A =$

40. Figure 18-12 shows the relationship of gears in a lathe using a simple gear train. The proportion given is used for lathe thread cutting computations using simple gearing. The fixed stud gear and the spindle gear have the same number of teeth. Determine the missing values for each of the following problems.

$$\frac{N_L}{N_C} = \frac{T_S}{T_L}$$

where N_L = number of threads per inch on the lead screw
N_C = number of threads per inch to be cut
T_S = number of teeth on stud gear
T_L = number of teeth on lead screw gear

➤ **Note:** Intermediate gears only change direction.

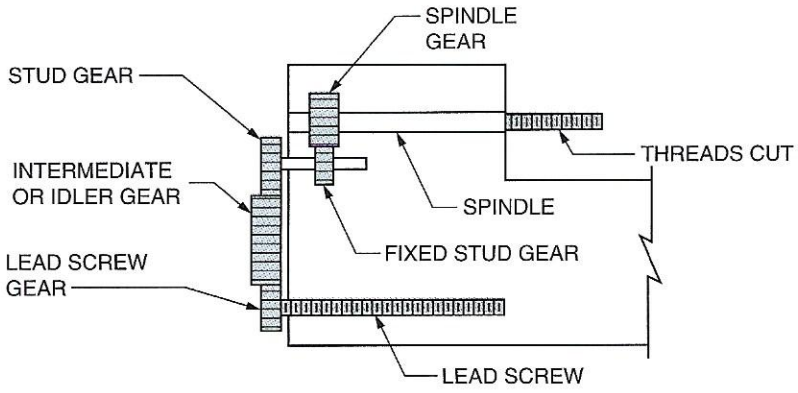


Figure 18-12

a. If $N_L = 4$, $N_C = 8$, and $T_S = 32$, find T_L. _____

b. If $N_L = 7$, $T_S = 35$, and $N_C = 15$, find T_L. _____

c. If $N_C = 10$, $N_L = 6$, and $T_L = 40$, find T_S. _____

d. If $N_L = 8$, $T_L = 42$, and $T_S = 28$, find N_C. _____

41. A template is shown on the left of Figure 18-13. A drafter makes an enlarged drawing of the template as shown on the right. The original length of 1.80 inches on the enlarged drawing is 3.06 inches as shown. Determine the lengths of A, B, C, and D.

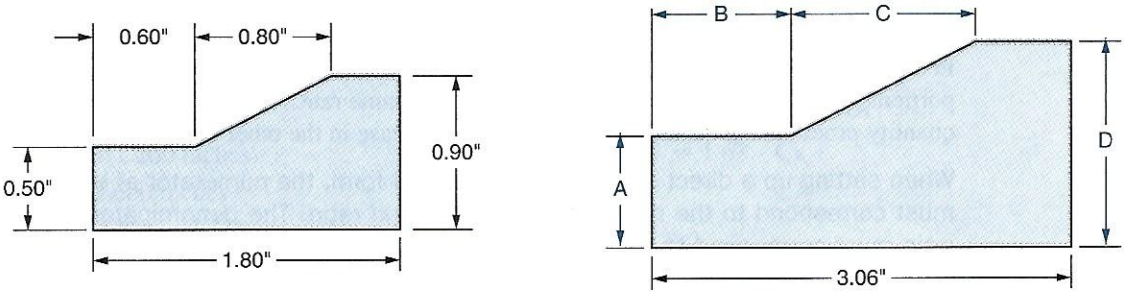


Figure 18-13

A _____ B _____ C _____ D _____

42. A drawing has a scale of $\frac{1}{4}" = 3'$. Determine the value of each missing value in the table.

	Scale Length	Actual Length
a.	$1\frac{1}{8}"$	
b.	$\frac{7}{16}"$	
c.		6' 9"
d.		$16'\ 1\frac{1}{16}"$